5c Applications

The ideas presented in Section 2 are now applied to basic percent problems and problems involving percent increase or decrease.

Basic Percent Problems

**EXAMPLE 1.** Misty answered 90% of the questions on her mathematics examination correctly. If Misty had 27 correct answers, how many questions were on the exam?

**Solution.** Let \( N \) represent the number of questions on the examination.

<table>
<thead>
<tr>
<th>Number of correct answers</th>
<th>is 90% of total number of questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>90% ( \cdot ) ( N )</td>
</tr>
</tbody>
</table>

Because \( 90\% = 0.90 \), this last equation can be written as

\[
27 = 0.90N.
\]

Solve for \( N \).

\[
\frac{27}{0.90} = \frac{0.90N}{0.90} \quad \text{Divide both sides by 0.90.}
\]

\[
30 = N \quad \text{Divide: } 27/0.90 = 30.
\]

Answer: 40

Hence, there were 30 questions on the examination.

**EXAMPLE 2.** 35 millilitres of a 60 millilitre solution is hydrochloric acid. What percent of the solution is hydrochloric acid?

**Solution.** Let \( p \) represent the percent of the percent of the solution that is hydrochloric acid. Then we can translate the problem statement into words and symbols.

<table>
<thead>
<tr>
<th>Amount of hydrochloric acid</th>
<th>is what percent of the total amount of solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>( p \cdot ) ( 60 )</td>
</tr>
</tbody>
</table>

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.

\[
35 = 60p
\]
Now we can solve for $p$.

\[
\frac{35}{60} = \frac{60p}{60} \quad \text{Divide both sides by 60.}
\]
\[
\frac{7}{12} = p \quad \text{Reduce: Divide numerator and denominator by 5.}
\]

Now we must change $p$ to a percent. We can do this exactly by creating an equivalent fraction with a denominator of 100.

\[
\frac{7}{12} = \frac{n}{100}
\]

Solve for $n$.

\[
12n = 700 \quad \text{Cross multiply.}
\]
\[
\frac{12n}{12} = \frac{700}{12} \quad \text{Divide both sides by 12.}
\]
\[
n = \frac{175}{3} \quad \text{Reduce: Divide numerator and denominator by 4.}
\]
\[
n = 58\frac{1}{3} \quad \text{Change improper to mixed fraction.}
\]

Hence,

\[
p = \frac{7}{12} = \frac{58\frac{4}{3}}{100} = 58\frac{1}{3}\%.
\]

Thus, $58\frac{1}{3}\%$ of the solution is hydrochloric acid.

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**Percent Increase or Decrease**

A person's salary can increase by a percentage. A town's population can decrease by a percentage. A clothing firm can discount its apparel. These are the types of applications we will now review.

**EXAMPLE 3.** A salesperson making a salary of $4,500 per month has his salary increased to $5,000 per month. What is the percent increase?

**Solution.** To find the increase in salary, first subtract the original salary from the new salary.

\[
\text{Salary increase} = \text{new salary} - \text{original salary} = 5000 - 4500 = 500
\]

A statistician making a salary of $3,200 per month has his salary increased to $3,368 per month. What is the percent increase?
Hence, the salesperson sees an increase in salary of $500.

Next, let $p$ represent the salesperson’s percent salary increase. Then we can translate the problem into words and symbols.

<table>
<thead>
<tr>
<th>Salary increase</th>
<th>is what percent of original salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>$p$</td>
</tr>
</tbody>
</table>

The commutative property of multiplication allows us to change the order of multiplication on the right-hand side of this last equation.

$$500 = 4500p$$

Solve for $p$.

$$\frac{500}{4500} = \frac{4500p}{4500}$$

Divide both sides by 4500.

$$\frac{1}{9} = p$$

Reduce by dividing numerator and denominator of 500/4500 by 500.

We need to change $p = 1/9$ to a percent. We can find an exact answer by creating an equivalent fraction with a denominator of 100.

$$\frac{1}{9} = \frac{n}{100}$$

Make equivalent fraction.

$$9n = 100$$

Cross multiply.

$$\frac{9n}{9} = \frac{100}{9}$$

Divide both sides by 9.

$$n = 11\frac{1}{9}$$

Convert 100/9 to mixed fraction.

Hence, the percent increase is

$$p = \frac{1}{9} = \frac{11\frac{1}{9}}{100} = 11\frac{1}{9} \%. \qquad \blacksquare$$

### You Try It!

A textile mill closure results in the population of the adjacent town decreasing from 8,956 to 7,800. What is the percent decrease in the population, rounded to the nearest tenth of a percent?

**Example 4.** Millertown falls on hard times and its population decreases from 11,256 to 10,923 in the space of one year. What is the percent decrease, rounded to the nearest hundredth of a percent?

**Solution.** To find the decrease in population, first subtract the current population from the original population.

$$\text{Population decrease} = \text{original population} - \text{current population}$$

$$= 11256 - 10923$$

$$= 333$$
Hence, the population has decreased by 333 people.

Next, let \( p \) represent the percent population decrease. Then we can translate the problem into words and symbols.

\[
\text{Population decrease} \quad \text{is} \quad \frac{\text{what percent}}{\text{of}} \quad \text{original population}
\]

\[
\begin{array}{cccc}
333 & = & p & \times 11256
\end{array}
\]

Solve for \( p \).

\[
\begin{array}{ccc}
\frac{333}{11256} & = & \frac{11256p}{11256} \\
0.02958 & \approx & p
\end{array}
\]

Divide both sides by 11256.

Divide: \( 333/11256 \approx 0.02958 \).

To change \( p \) to a percent, move the decimal point two places to the right and append a percent symbol.

\[
p \approx 0.02958 \approx 0.02958\% \approx 2.958\%.
\]

We are asked to round to the nearest hundredth of a percent.

\[
p \approx 2.96\%.
\]

Thus, the population of Millertown decreases approximately 2.96%.

\[
\square
\]

**EXAMPLE 5.** A pair of skis is marked at \$310. However, a sign in the shop indicates that skis are being discounted at 15\%. What will be the new selling price of the skis?

**Solution.** Let \( D \) represent the discount (in dollars) given for our pair of skis. Then, in words and symbols:

\[
\begin{array}{ccc}
\text{Discount} & \text{is} & \frac{15\%}{\text{of}} \quad \text{original marked price}
\end{array}
\]

\[
D = 15\% \times 310
\]

A pair of hiking boots is marked at \$200. During a sale, the boots are discounted by 8\%. What is the new price of the boots?
Solve for $D$.

$$D = 0.15 \cdot 310$$
$$D = 46.50$$

Hence, the discount is $46.50$. To find the new selling price, subtract this discount from the original selling price.

New selling price = original selling price – discount

$$= 310 - 46.50$$
$$= 263.50$$

Hence, the new selling price is $263.50$.

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**EXAMPLE 6.** A pair of ski boots marked at $210 is sold for $180. Find the percent discount, correct to the nearest tenth of a percent.

**Solution.** We can find the discount (in dollars) by subtracting the sale price from the original marked price.

Discount = original marked price – sale price

$$= 210 - 180$$
$$= 30$$

Hence, the boots are discounted $30.

Let $p$ represent the percent discount. Then, in words and symbols:

\[
\text{Discount is percent discount of original marked price} \quad \frac{30}{210} = p
\]

Solve for $p$.

$$30 = 210p$$
$$\frac{30}{210} = \frac{210p}{210}$$
$$\frac{1}{7} = p$$

Reduce: Divide numerator and denominator of $30/210$ by 30.

$$p \approx 0.1428$$

Divide: $1/7 \approx 0.1428$.

To change $p$ to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 0.1428 \approx 0.14.28% \approx 14.28\%.$$

Round to the nearest tenth of a percent.
Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate. Thus, correct to the nearest tenth of a percent, the percent discount is

\[ p \approx 14.3\% . \]